

Sum and Difference Identities

Find the exact value of each.

1) $\sin -15$

2) $\sin 165$

3) $\cos \frac{17\pi}{12}$

4) $\cos -105$

5) $\tan 105$

6) $\tan \frac{5\pi}{12}$

7)
$$\frac{\tan \frac{8\pi}{9} - \tan \frac{\pi}{18}}{1 + \tan \frac{8\pi}{9} \tan \frac{\pi}{18}}$$

8) $\cos 10\cos 35 - \sin 10\sin 35$

9)
$$\frac{\tan 76 + \tan 164}{1 - \tan 76 \tan 164}$$

10)
$$\sin \frac{\pi}{9} \cos \frac{2\pi}{9} + \cos \frac{\pi}{9} \sin \frac{2\pi}{9}$$

Simplify.

11) $\sin -6\theta\cos 4\theta + \cos -6\theta\sin 4\theta$

12) $\sin \theta\cos 3\theta + \cos \theta\sin 3\theta$

13) $\cos 3x\cos -4x + \sin 3x\sin -4x$

14) $\frac{\tan u + \tan 4u}{1 - \tan u \tan 4u}$

Write each trigonometric expression as an algebraic expression.

15) $\sin\left(\cos^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}2x\right)$

16) $\sin(\sin^{-1}4x + \cos^{-1}x)$

17) $\cos(\tan^{-1}2x + \sin^{-1}0)$

18) $\tan(\tan^{-1}1 - \tan^{-1}\sqrt{x})$

Verify each identity.

19) $\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$

20) $\cos(\pi + \theta) = -\cos \theta$

Sum and Difference Identities

Find the exact value of each.

1) $\sin -15$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

2) $\sin 165$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

3) $\cos \frac{17\pi}{12}$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

4) $\cos -105$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

5) $\tan 105$

$$-2 - \sqrt{3}$$

6) $\tan \frac{5\pi}{12}$

$$2 + \sqrt{3}$$

7)
$$\frac{\tan \frac{8\pi}{9} - \tan \frac{\pi}{18}}{1 + \tan \frac{8\pi}{9} \tan \frac{\pi}{18}}$$

$$-\frac{\sqrt{3}}{3}$$

8) $\cos 10\cos 35 - \sin 10\sin 35$

$$\frac{\sqrt{2}}{2}$$

9)
$$\frac{\tan 76 + \tan 164}{1 - \tan 76 \tan 164}$$

$$\sqrt{3}$$

10)
$$\sin \frac{\pi}{9} \cos \frac{2\pi}{9} + \cos \frac{\pi}{9} \sin \frac{2\pi}{9}$$

$$\frac{\sqrt{3}}{2}$$

Simplify.

11) $\sin -6\theta \cos 4\theta + \cos -6\theta \sin 4\theta$

$\sin -2\theta$

12) $\sin \theta \cos 3\theta + \cos \theta \sin 3\theta$

$\sin 4\theta$

13) $\cos 3x \cos -4x + \sin 3x \sin -4x$

$\cos 7x$

14) $\frac{\tan u + \tan 4u}{1 - \tan u \tan 4u}$

$\tan 5u$

Write each trigonometric expression as an algebraic expression.

15) $\sin \left(\cos^{-1} \frac{\sqrt{2}}{2} - \sin^{-1} 2x \right)$

$$\frac{-2x\sqrt{2} + \sqrt{2 - 8x^2}}{2}$$

16) $\sin (\sin^{-1} 4x + \cos^{-1} x)$

$4x^2 + \sqrt{1 - 17x^2 + 16x^4}$

17) $\cos (\tan^{-1} 2x + \sin^{-1} 0)$

$$\frac{\sqrt{4x^2 + 1}}{4x^2 + 1}$$

18) $\tan (\tan^{-1} 1 - \tan^{-1} \sqrt{x})$

$$\frac{1 - 2\sqrt{x} + x}{1 - x}$$

Verify each identity.

19) $\sin \left(\frac{\pi}{2} + \theta \right) = \cos \theta$

$\sin \left(\frac{\pi}{2} + \theta \right)$

$= \sin \frac{\pi}{2} \cos \theta + \cos \frac{\pi}{2} \sin \theta$

$= \cos \theta + 0 \sin \theta$

$= \cos \theta$

20) $\cos (\pi + \theta) = -\cos \theta$

$\cos (\pi + \theta)$

$= \cos \pi \cos \theta - \sin \pi \sin \theta$

$= -\cos \theta - 0 \sin \theta$

$= -\cos \theta$