A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

1) \(-1, \ 1 + 3i\)  
2) \(-\frac{1}{4}, \ 1 + \sqrt{6}\)  
3) \(-3\) mult. 2, \(2\sqrt{2}\)  
4) \(1 + \sqrt{3}, \ -3 + \sqrt{5}\)  
5) \(1 - i, \ \sqrt{7}\)  
6) \(-3 + 2i, \ -2 - 2i, \ -2 + 2i\)

Write a polynomial function of least degree with integral coefficients that has the given zeros.

7) \(-\frac{1}{2}, \ 1, \ \frac{3}{4}\)  
8) \(-1, \ -i\)  
9) 2 mult. 3  
10) \(-3, \ 2\sqrt{2}\)  
11) \(-3, \ \sqrt{3}\)  
12) \(1 + \sqrt{10}\) mult. 2, \(1 - \sqrt{10}\)  
13) \(-i\) mult. 2  
14) \(\frac{4}{5}, \ 2i\)

Critical thinking questions:

15) Explain why it makes sense that a third-degree polynomial must have at least one rational zero.  
16) Write a polynomial function of degree ten that has two imaginary roots.
Polynomials and Conjugate Roots

A polynomial function with rational coefficients has the following zeros. Find all additional zeros.

1) \(-1, \ 1 + 3i\)  \(1 - 3i\)
2) \(-\frac{1}{4}, \ 1 + \sqrt{6}\)  \(1 - \sqrt{6}\)

3) \(-3\) mult. 2, \(2\sqrt{2}\)  \(-2\sqrt{2}\)
4) \(1 + \sqrt{3}, \ -3 + \sqrt{5}\)  \(1 - \sqrt{3}, \ -3 - \sqrt{5}\)

5) \(1 - i, \ \sqrt{7}\)  \(1 + i, \ -\sqrt{7}\)
6) \(-3 + 2i, \ -2 - 2i, \ -2 + 2i\)  \(-3 - 2i\)

Write a polynomial function of least degree with integral coefficients that has the given zeros.

7) \(-\frac{1}{2}, \ 1, \ \frac{3}{4}\)
\(f(x) = 8x^3 - 10x^2 - x + 3\)

8) \(-1, \ -i\)
\(f(x) = x^3 + x^2 + x + 1\)

9) 2 mult. 3
\(f(x) = x^3 - 6x^2 + 12x - 8\)

10) \(-3, \ 2\sqrt{2}\)
\(f(x) = x^3 + 3x^2 - 8x - 24\)

11) \(-3, \ \sqrt{3}\)
\(f(x) = x^3 + 3x^2 - 3x - 9\)

12) \(1 + \sqrt{10}\) mult. 2, \(1 - \sqrt{10}\)
\(f(x) = x^4 - 4x^3 - 14x^2 + 36x + 81\)

13) \(-i\) mult. 2
\(f(x) = x^4 + 2x^2 + 1\)

14) \(\frac{4}{5}, \ 2i\)
\(f(x) = 5x^3 - 4x^2 + 20x - 16\)

Critical thinking questions:

15) Explain why it makes sense that a third-degree polynomial must have at least one rational zero. It must go from \(+\infty\) to \(-\infty\) so it must cross the x-axis.

16) Write a polynomial function of degree ten that has two imaginary roots.
\(f(x) = (x^2 + 1)^5\)