

Differentials

For each problem, find the differential dy .

1) $y = -x^3 - 2$

2) $y = -\frac{3}{x}$

For each problem, find the general formulas for dy and Δy .

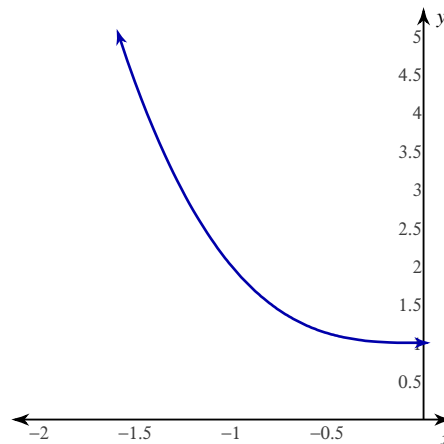
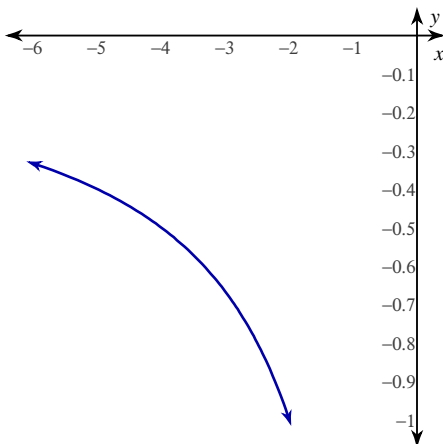
3) $y = -x^3 - 2$

4) $y = \frac{2}{x}$

For each problem, find dy and Δy , given x_0 and $dx = \Delta x$. You may use the provided graph of the function to sketch dx , Δx , dy , and Δy .

5) $y = \frac{2}{x}$; $x_0 = -5$, $dx = \Delta x = \frac{5}{2}$

6) $y = -x^3 + 1$; $x_0 = -1$, $dx = \Delta x = -\frac{1}{2}$



For each problem, find a linear approximation of the given quantity.

7) $\sin 122^\circ$

8) 6.99^4

Use differentials to solve each problem.

9) The radius of a sphere is measured to be 7 cm, with a possible error of $\pm \frac{1}{10}$ cm. Estimate the possible propagated error in the calculated volume.

10) The sides of a square are measured to be 4 in, with a possible error of $\pm \frac{1}{5}$ in. Estimate the possible propagated error in the calculated area.

Differentials

For each problem, find the differential dy .

1) $y = -x^3 - 2$

$$dy = -3x^2 dx$$

2) $y = -\frac{3}{x}$

$$dy = \frac{3}{x^2} dx$$

For each problem, find the general formulas for dy and Δy .

3) $y = -x^3 - 2$

$$dy = -3x^2 dx$$

$$\Delta y = -3x^2 \Delta x - 3x(\Delta x)^2 - (\Delta x)^3$$

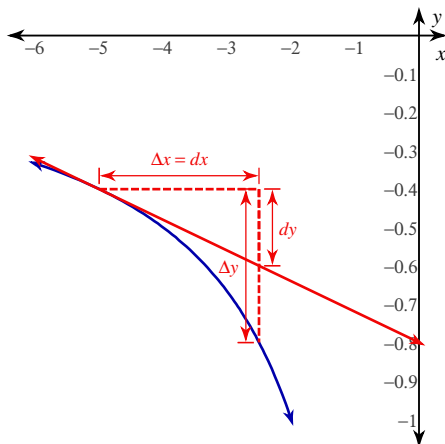
4) $y = \frac{2}{x}$

$$dy = -\frac{2}{x^2} dx$$

$$\Delta y = -\frac{2\Delta x}{x^2 + x\Delta x}$$

For each problem, find dy and Δy , given x_0 and $dx = \Delta x$. You may use the provided graph of the function to sketch dx , Δx , dy , and Δy .

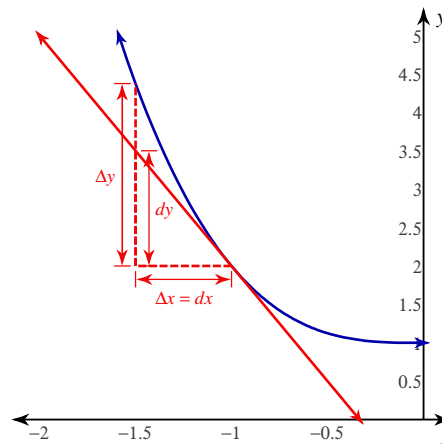
5) $y = \frac{2}{x}$; $x_0 = -5$, $dx = \Delta x = \frac{5}{2}$



$$dy = -\frac{1}{5} = -0.2$$

$$\Delta y = -\frac{2}{5} = -0.4$$

6) $y = -x^3 + 1$; $x_0 = -1$, $dx = \Delta x = -\frac{1}{2}$



$$dy = \frac{3}{2} = 1.5$$

$$\Delta y = \frac{19}{8} = 2.375$$

For each problem, find a linear approximation of the given quantity.

7) $\sin 122^\circ$

$$f(x) = \sin x, f'(x) = \cos x$$

$$x_0 = \frac{2\pi}{3} \text{ radians}, \Delta x = \frac{\pi}{90} \text{ radians}$$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{90\sqrt{3} - \pi}{180} \approx 0.8486$$

8) 6.99^4

$$f(x) = x^4, f'(x) = 4x^3$$

$$x_0 = 7, \Delta x = -0.01$$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{59682}{25} = 2387.28$$

Use differentials to solve each problem.

- 9) The radius of a sphere is measured to be 7 cm, with a possible error of $\pm \frac{1}{10}$ cm. Estimate the possible propagated error in the calculated volume.

$$V = \frac{4}{3}\pi r^3, dV = 4\pi r^2 dr$$

$$r = 7, dr = \pm 0.1$$

$$\Delta V \approx dV = \pm \frac{98\pi}{5} \approx \pm 61.5752 \text{ cm}^3$$

- 10) The sides of a square are measured to be 4 in, with a possible error of $\pm \frac{1}{5}$ in. Estimate the possible propagated error in the calculated area.

$$A = s^2, dA = 2s ds$$

$$s = 4, ds = \pm 0.2$$

$$\Delta A \approx dA = \pm \frac{8}{5} = \pm 1.6 \text{ in}^2$$