

Analyzing and Solving Polynomial Equations

Date _____ Period _____

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$

2) $x^3 + 3x^2 - 14x - 20 = 0$

3) $x^3 - 2x^2 + 3x - 6 = 0$

4) $x^4 - 14x^2 + 45 = 0$

5) $x^4 + 6x^2 + 8 = 0$

6) $x^4 + 3x^2 - 18 = 0$

7) $x^3 - 1 = 0$

8) $x^3 + 3x^2 - x - 3 = 0$

$$9) x^3 - 2x^2 - 3x + 6 = 0$$

$$10) x^6 - 2x^4 - 4x^2 + 8 = 0$$

$$11) x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$$

$$12) x^6 + 5x^4 - 4x^2 - 20 = 0$$

$$13) x^6 - x^4 - x^2 + 1 = 0$$

$$14) x^8 - 26x^4 + 25 = 0$$

Analyzing and Solving Polynomial Equations

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$

of complex roots: 4
 Possible # of real roots: 4, 2, or 0
 Possible # of imaginary roots: 4, 2, or 0
 Possible # positive real roots: 1
 Possible # negative real roots: 1
 Possible rational roots:
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$
 Roots: $\{2i, -2i, 3, -3\}$

2) $x^3 + 3x^2 - 14x - 20 = 0$

of complex roots: 3
 Possible # of real roots: 3 or 1
 Possible # of imaginary roots: 2 or 0
 Possible # positive real roots: 1
 Possible # negative real roots: 2 or 0
 Possible rational roots:
 $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
 Roots: $\{-5, 1 + \sqrt{5}, 1 - \sqrt{5}\}$

3) $x^3 - 2x^2 + 3x - 6 = 0$

of complex roots: 3
 Possible # of real roots: 3 or 1
 Possible # of imaginary roots: 2 or 0
 Possible # positive real roots: 3 or 1
 Possible # negative real roots: 0
 Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$
 Roots: $\{2, i\sqrt{3}, -i\sqrt{3}\}$

4) $x^4 - 14x^2 + 45 = 0$

of complex roots: 4
 Possible # of real roots: 4, 2, or 0
 Possible # of imaginary roots: 4, 2, or 0
 Possible # positive real roots: 2 or 0
 Possible # negative real roots: 2 or 0
 Possible rational roots:
 $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$
 Roots: $\{\sqrt{5}, -\sqrt{5}, 3, -3\}$

5) $x^4 + 6x^2 + 8 = 0$

of complex roots: 4
 Possible # of real roots: 4, 2, or 0
 Possible # of imaginary roots: 4, 2, or 0
 Possible # positive real roots: 0
 Possible # negative real roots: 0
 Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8$
 Roots: $\{2i, -2i, i\sqrt{2}, -i\sqrt{2}\}$

6) $x^4 + 3x^2 - 18 = 0$

of complex roots: 4
 Possible # of real roots: 4, 2, or 0
 Possible # of imaginary roots: 4, 2, or 0
 Possible # positive real roots: 1
 Possible # negative real roots: 1
 Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
 Roots: $\{\sqrt{3}, -\sqrt{3}, i\sqrt{6}, -i\sqrt{6}\}$

7) $x^3 - 1 = 0$

of complex roots: 3
 Possible # of real roots: 3 or 1
 Possible # of imaginary roots: 2 or 0
 Possible # positive real roots: 1
 Possible # negative real roots: 0
 Possible rational roots: ± 1
 Roots: $\left\{1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}\right\}$

8) $x^3 + 3x^2 - x - 3 = 0$

of complex roots: 3
 Possible # of real roots: 3 or 1
 Possible # of imaginary roots: 2 or 0
 Possible # positive real roots: 1
 Possible # negative real roots: 2 or 0
 Possible rational roots: $\pm 1, \pm 3$
 Roots: $\{-3, 1, -1\}$

$$9) x^3 - 2x^2 - 3x + 6 = 0$$

of complex roots: 3
 Possible # of real roots: 3 or 1
 Possible # of imaginary roots: 2 or 0
 Possible # positive real roots: 2 or 0
 Possible # negative real roots: 1
 Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$
 Roots: $\{2, \sqrt{3}, -\sqrt{3}\}$

$$10) x^6 - 2x^4 - 4x^2 + 8 = 0$$

of complex roots: 6
 Possible # of real roots: 6, 4, 2, or 0
 Possible # of imaginary roots: 6, 4, 2, or 0
 Possible # positive real roots: 2 or 0
 Possible # negative real roots: 2 or 0
 Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8$
 Roots: $\{\sqrt{2} \text{ mult. } 2, -\sqrt{2} \text{ mult. } 2, i\sqrt{2}, -i\sqrt{2}\}$

$$11) x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$$

of complex roots: 5
 Possible # of real roots: 5, 3, or 1
 Possible # of imaginary roots: 4, 2, or 0
 Possible # positive real roots: 0
 Possible # negative real roots: 5, 3, or 1
 Possible rational roots:
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$
 Roots: $\{-2, i\sqrt{3}, -i\sqrt{3}, 2i\sqrt{2}, -2i\sqrt{2}\}$

$$12) x^6 + 5x^4 - 4x^2 - 20 = 0$$

of complex roots: 6
 Possible # of real roots: 6, 4, 2, or 0
 Possible # of imaginary roots: 6, 4, 2, or 0
 Possible # positive real roots: 1
 Possible # negative real roots: 1
 Possible rational roots:
 $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
 Roots: $\{i\sqrt{5}, -i\sqrt{5}, \sqrt{2}, -\sqrt{2}, i\sqrt{2}, -i\sqrt{2}\}$

$$13) x^6 - x^4 - x^2 + 1 = 0$$

of complex roots: 6
 Possible # of real roots: 6, 4, 2, or 0
 Possible # of imaginary roots: 6, 4, 2, or 0
 Possible # positive real roots: 2 or 0
 Possible # negative real roots: 2 or 0
 Possible rational roots: ± 1
 Roots: $\{1 \text{ mult. } 2, -1 \text{ mult. } 2, i, -i\}$

$$14) x^8 - 26x^4 + 25 = 0$$

of complex roots: 8
 Possible # of real roots: 8, 6, 4, 2, or 0
 Possible # of imaginary roots: 8, 6, 4, 2, or 0
 Possible # positive real roots: 2 or 0
 Possible # negative real roots: 2 or 0
 Possible rational roots: $\pm 1, \pm 5, \pm 25$
 Roots: $\{1, -1, i, -i, \sqrt{5}, -\sqrt{5}, i\sqrt{5}, -i\sqrt{5}\}$